$$\begin{cases} \partial_t f & = \sigma_x \Delta_x f - \mathrm{Pe} \nabla_x \cdot (v(\theta) f) + \sigma_\theta \partial_\theta^2 f - \chi \partial_\theta (n(\theta) \cdot (\nabla K * \rho + \tau \nabla^2 K * \rho v(\theta)) f) \\ f(t=0) & = f_0 \end{cases}$$

$$\begin{cases} \mathrm{d}X_t^i &= \mathrm{Pe}v(\theta_t^i)\mathrm{d}t + \sqrt{2\sigma_x}\mathrm{d}W_t^i \\ \mathrm{d}\theta_t^i &= \chi n(\theta_t^i) \cdot \frac{1}{N}\sum_{j \neq i} \nabla K(X_t^i + \tau v(\theta_t^i) - X_t^j) + \sqrt{2}\mathrm{d}B_t^i \end{cases}$$

- Let $F_s = \{f_s(x, \theta)\}$ be translates of the spot solution and F_l the translates of the lane solution

- If $f_0 \in F_s \cup F_l$ then $f_t = f_0$ for all t by stationarity

- **Open**: by fluctuations, μ_t^N will mostly be in the basin of attraction of the stable solution for any initial data, so that $\lim_{N\to\infty} \lim_{t\to\infty} \mu_t^N \in F_{\text{stable}}$ even if $\lim_{t\to\infty} \lim_{N\to\infty} f_t = f_0 \in F_{\text{unstable}}$?

Dynamics: non-uniform convergence?

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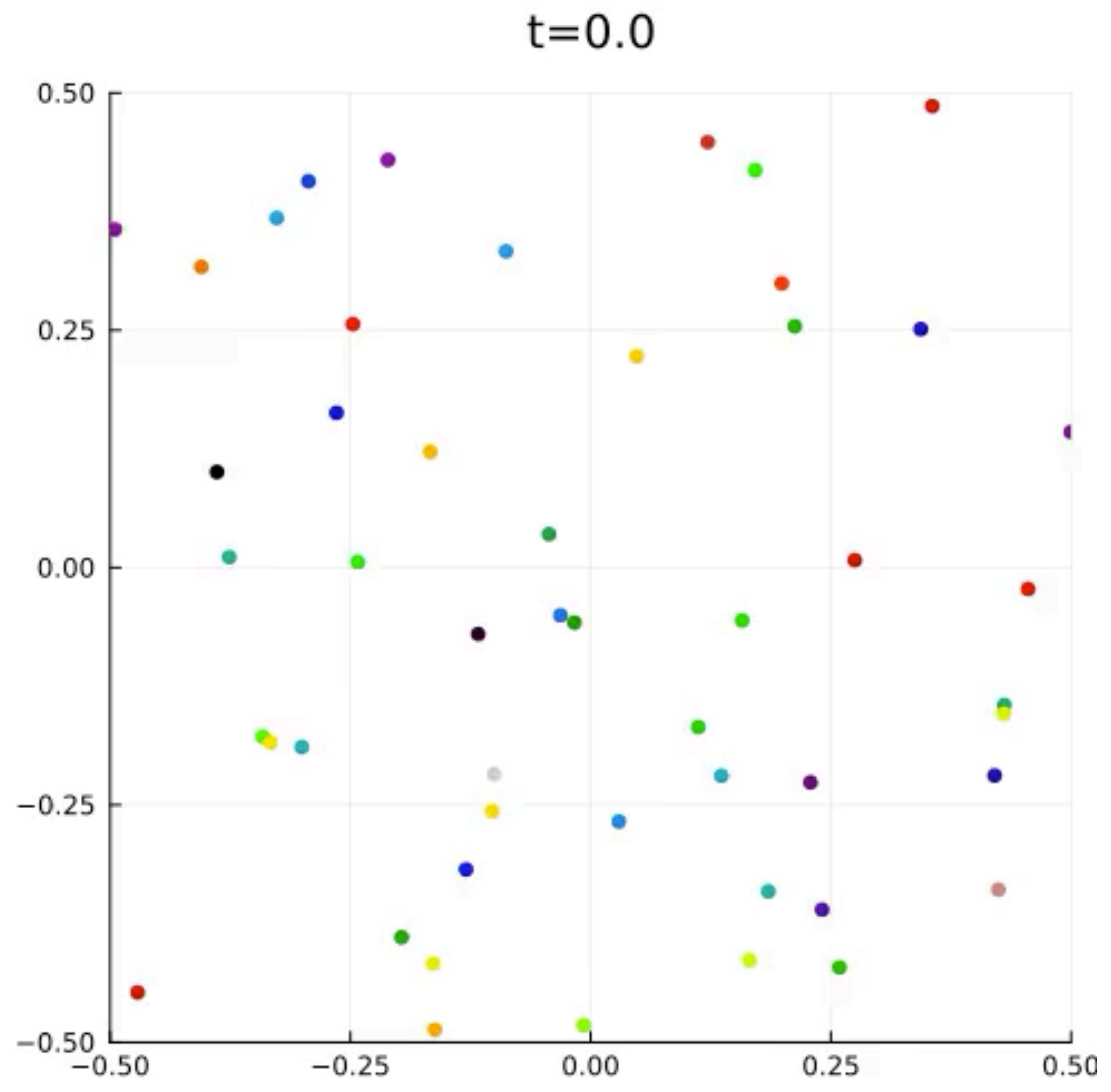
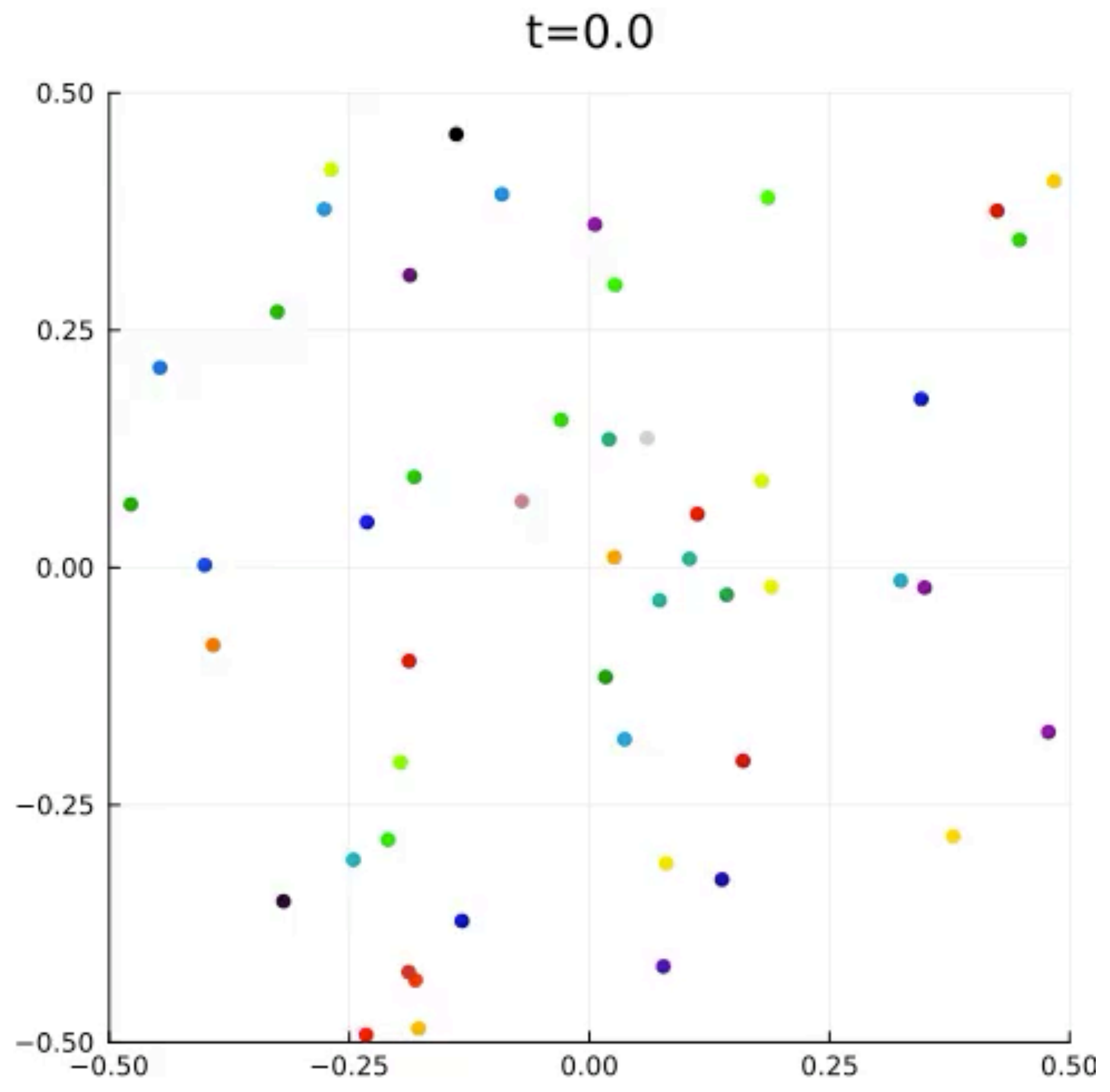
Dynamics





$\tau = 0$

$\tau = 0.15$