

$$\begin{cases} \partial_t f = \sigma_x \Delta_x f - \text{Pe} \nabla_x \cdot (v(\theta) f) + \partial_\theta^2 f - \chi \partial_\theta (n(\theta) \cdot (\nabla K^* \rho)_\tau f) \\ f(t=0) = f_0 \end{cases}$$

(dV, Brunna & Burger, SIAM J. Appl. Dyn. Syst., 2025):

- PDE always globally-in-time well-posed for singular Newtonian ∇K and $\sigma_x \geq 0$

• **No blow-up in finite time (Alfakros, CPDE, 1979)**

Open enrollment periods

- Propagation of chaos for singular ∇K and larger τ

- Propagation of chaos and global-in-time well-posedness for singular ∇K and $\sigma_x = 0$

(Chaintreuil & Diez, Kinet. Rel. Models, 2022)

Ant analysis

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$$\begin{cases} \partial_t f &= \sigma_x \Delta_x f - \text{Pe} \nabla_x \cdot (v(\theta)f) + \partial_\theta^2 f - \chi \partial_\theta (n(\theta) \cdot (\nabla K * \rho)_\tau f) \\ f(t=0) &= f_0 \end{cases}$$

(dW, Bruna & Burger, SIAM J. Appl. Dyn. Syst., 2025):

- PDE always globally-in-time well-posed for singular Newtonian ∇K and $\sigma_x > 0$
- No blow-up in infinite time (Alikakos, CPDE, 1979)

Open problems

- Propagation of chaos for singular ∇K and larger τ
- Propagation of chaos and global-in-time well-posedness for singular ∇K and $\sigma_x = 0$
 - (Chaintron & Diez, Kinet. Rel. Models, 2022)

Ant analysis

$$\begin{cases} \partial_t f &= \sigma_x \Delta_x f - \text{Pe} \nabla_x \cdot (v(\theta) f) + \partial_\theta^2 f - \chi \partial_\theta (n(\theta) \cdot (\nabla K * \rho)_\tau f) \\ f(t=0) &= f_0 \end{cases}$$

- What happens as $t \rightarrow \infty$?