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(dW, Bruna & Burger, SIAM J. Appl. Dyn. Syst., 2025):

- PDE always globally-in-time well-posed for singular Newtonian ∇K and $\sigma_x > 0$

- No blow-up in infinite time (Alikakos, CPDE, 1979)

Open problems

- Propagation of chaos for singular ∇K and larger τ

- Propagation of chaos and global-in-time well-posedness for singular ∇K and $\sigma_x = 0$

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- What happens as $t \to \infty$?