

Active ants: dynamics

$$\begin{cases} \partial_t f &= \sigma_x \Delta_x f - \text{Pe} \nabla_x \cdot (v(\theta) f) \\ &+ \sigma_\theta \partial_\theta^2 f - \chi \partial_\theta (n(\theta) \cdot (\nabla K * \rho + \tau \nabla^2 K * \rho v(\theta)) f) \\ f(t=0) &= f_0 \end{cases}$$

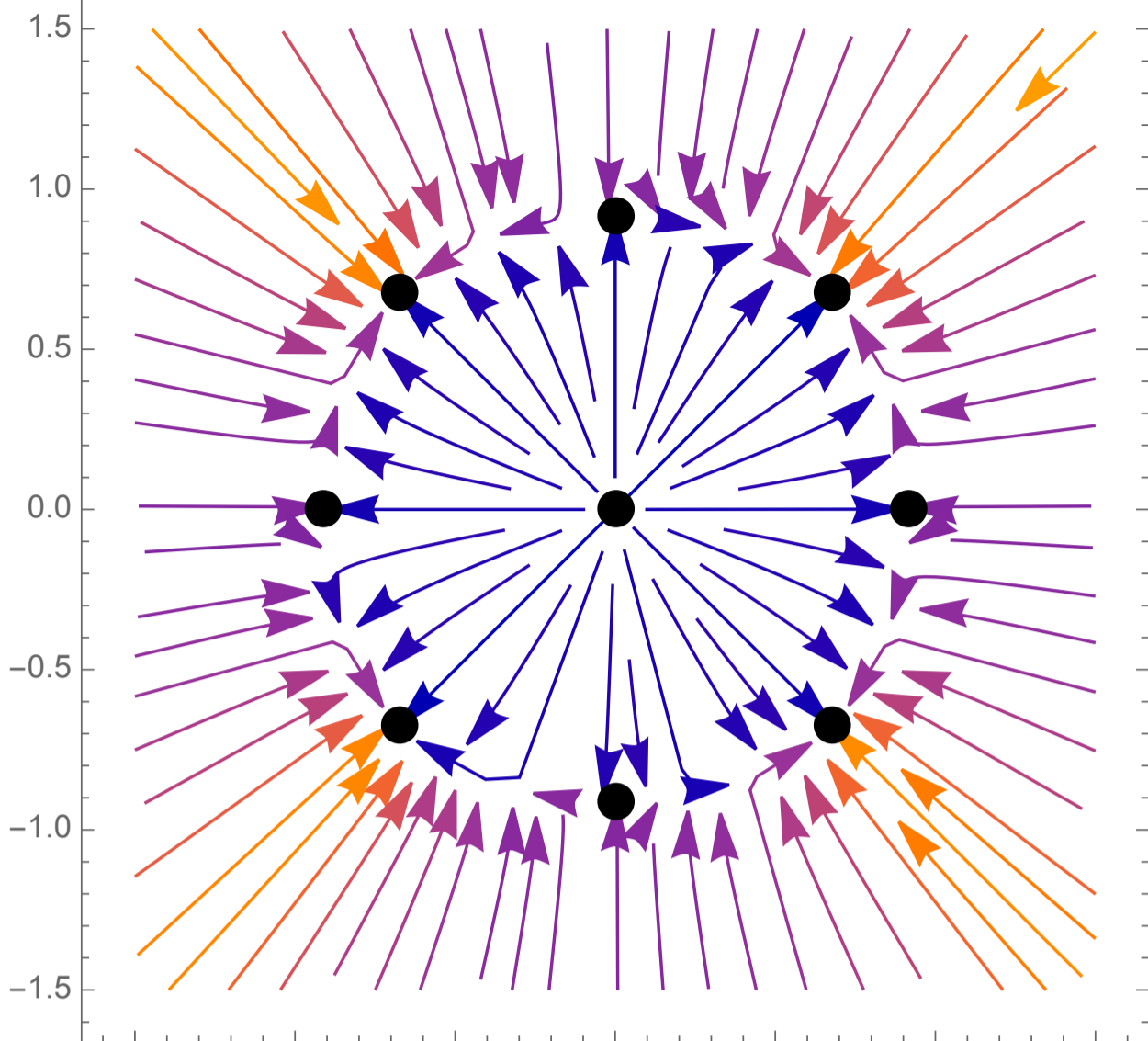
• What happens as $t \rightarrow \infty$?

• Part 2 (Rakotomalala & dVW, in review, 2026):

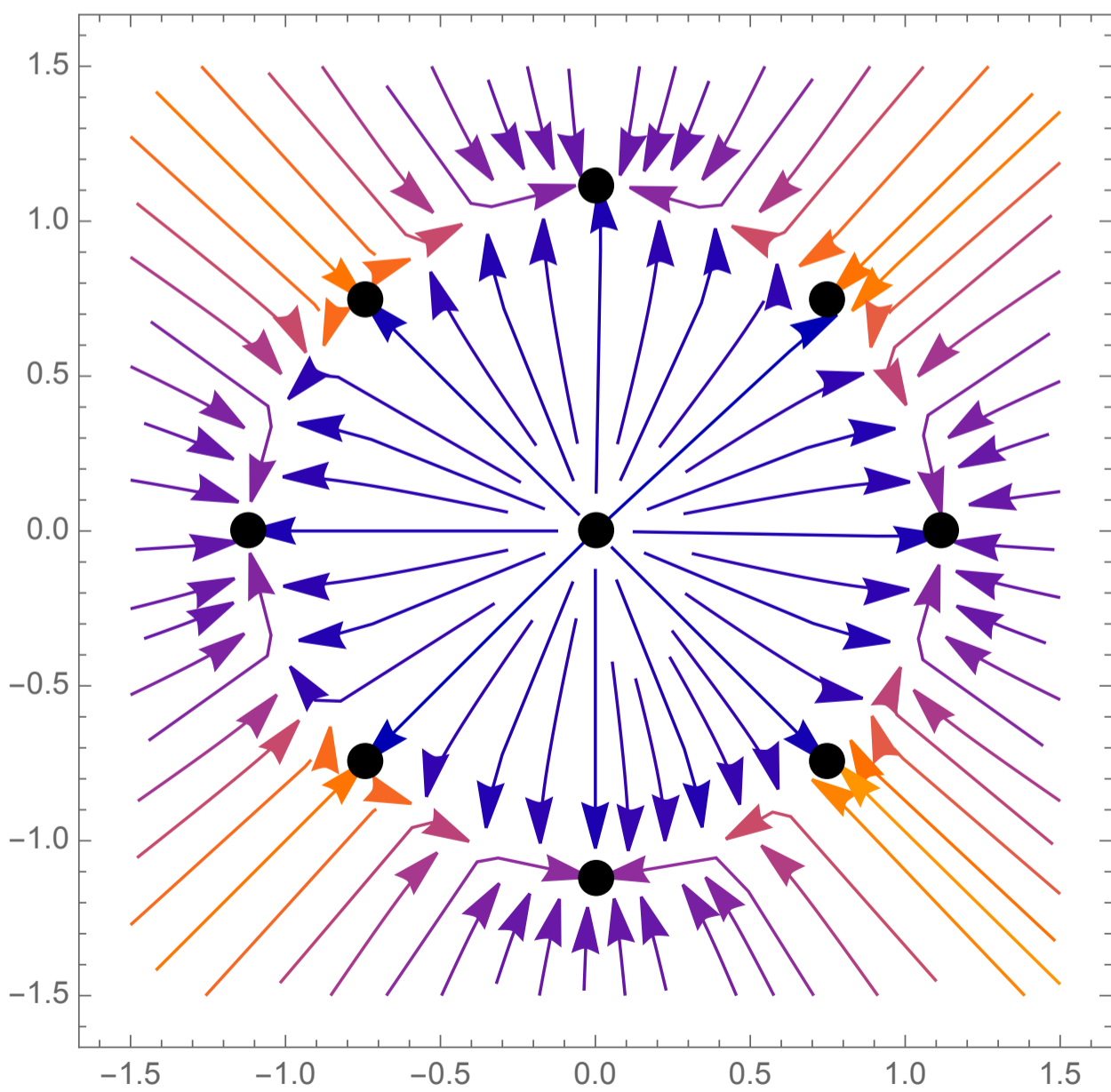
Existence of two families of non-trivial stationary states

◦ Existence of critical $\tau \equiv \tau^*$ super- to subcritical

Lyapunov-Schmidt method: PDE \Leftrightarrow ODE



$$\tau < \tau^*$$

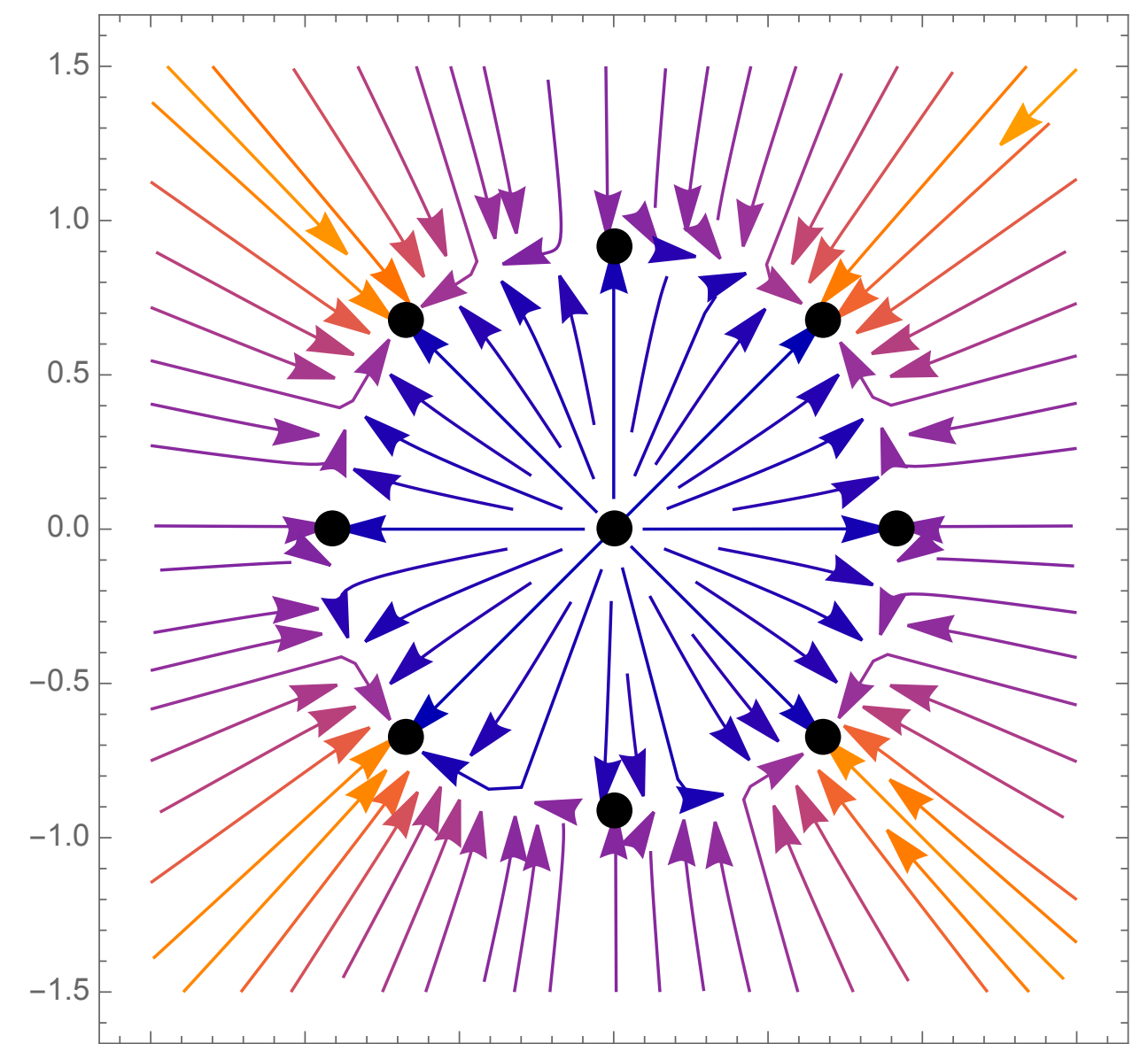


$$\tau > \tau^*$$

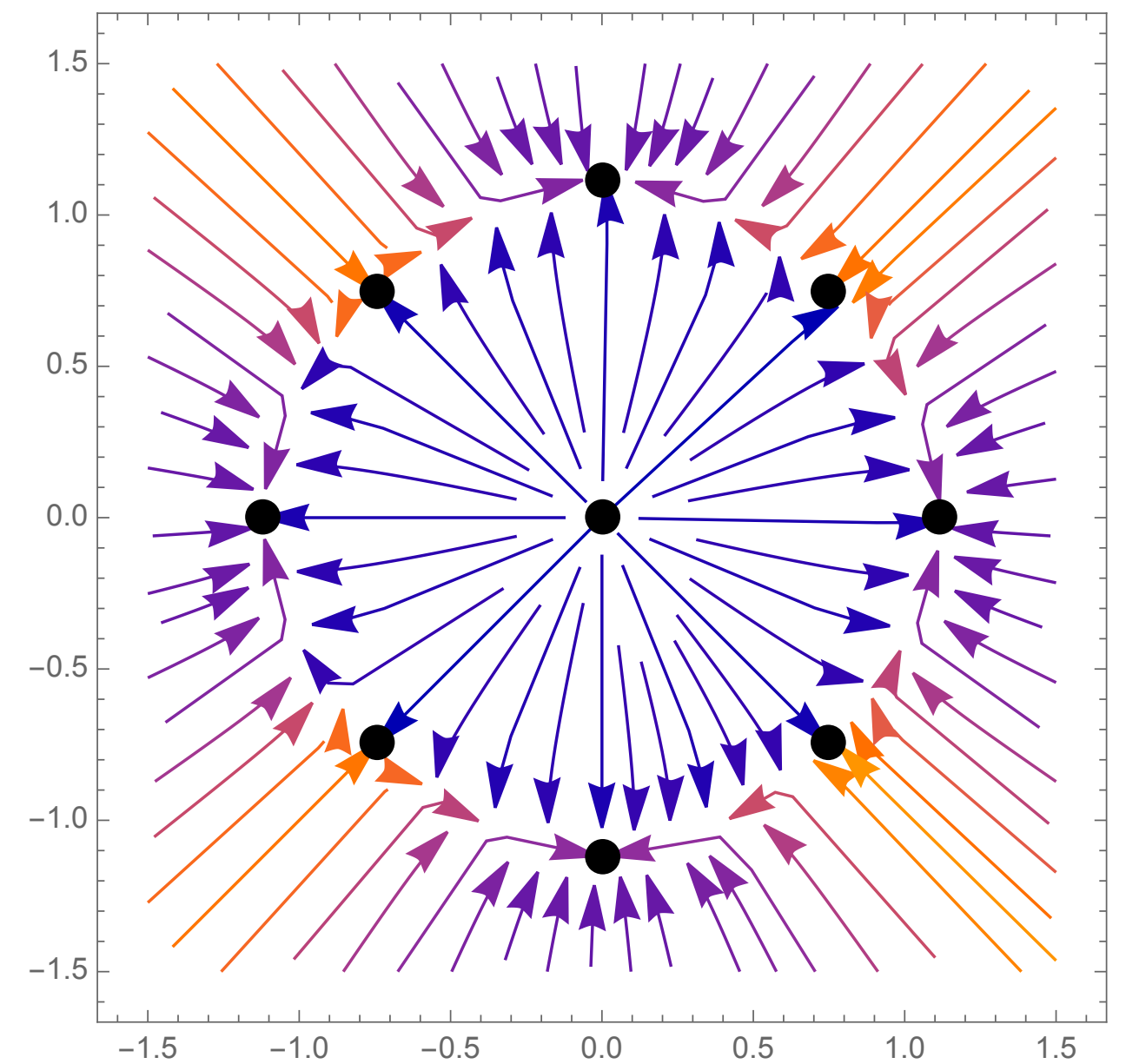
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- What happens as $t \rightarrow \infty$?
- Part 2 (Rakotomalala & dW, in review, 2026):
 - Existence of two families of non-trivial stationary states
 - Existence of critical $\tau = \tau^*$ super- to subcritical
 - Lyapunov-Schmidt method: PDE \iff ODE



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Lyapunov-Schmidt method